5. Inflation-linked bonds

Inflation is an economic term that describes the general rise in prices of goods and services. As prices rise, a unit of money can buy less goods and services. Hence, inflation is an important risk factor for investors, because it can erode the purchasing power of their investments. For example, suppose an investor acquires a one-year nominal bond at par value with an annual nominal rate of 5%. If annual inflation turns out to be 2%, then the investor's purchasing power increases by roughly 3%, but if actual inflation is instead 7%, then the investor's purchasing power would indeed decrease by 2%.

Inflation-linked bonds are securities that are designed to protect investors from this inflation risk. Like other bonds, they are issued with a fixed coupon rate and a fixed maturity date, but their payoffs are linked to inflation. So, if prices go up, payments of these bonds increase too, maintaining the purchasing power of its holders.

5.1 Real and break-even rates

Conventional bonds promise fixed nominal coupons and the redemption payment of the principal, but the real value, that is how many products and services can be bought with these payments when they are received, is unknown because it depends on the future inflation rate. On the contrary, the payments of inflation-linked bonds are indexed to inflation and therefore guarantee a fixed real return irrespective of inflation.

The yield of a conventional bond consists mainly of three parts, a real yield which is a compensation for delayed consumption and an expected inflation over the life of the bond, which is a compensation for the expected loss in purchasing power due to inflation. So, conventional bonds also compensate for inflation, but it is ex ante. If realized inflation turns out to be higher than expected, then the investor suffers a loss in purchasing power. Due to this risk, the third part of the yield of a conventional bond is an inflation risk premium to compensate the investor. So,

$$Nominal\ \ Yield = Real\ \ Yield + Expected\ \ Inflation\ \ Rate + Inflation\ \ Risk\ \ Premium$$ [21]

On the other hand, the yield of an inflation-linked bond consists only of two parts, a real yield and the realized inflation over the life of the bond, but as this inflation compensation is applied after issuance, ex post, the rate when the bond is issued consists only of the real yield.

$$Inflation\text{-}linked\ \ Yield = Real\ \ Yield + Actual\ \ Inflation\ \ Rate$$

[21] This formula is an approximation, the actual formula is $(1+Nominal\ \ Yield) = (1+Real\ \ Yield)\cdot(1+Expected\ \ Inflation\ \ Rate)\cdot(1+Inflation\ \ Risk\ \ Premium)$ This approximation is appropriate when yields and inflation are low and therefore is a general industry practice.

From the nominal yields of a conventional bond and of a comparable inflation-linked bond of the same maturity, an estimation of the inflation expected by the market over the term of the bonds can be obtained. This is called the breakeven inflation, and is calculated, ignoring the inflation risk premium[22], as:

$$Breakeven\ Inflation = Nominal\ \ Yield - Inflation\text{-}linked\ Yield$$ [23]

If actual inflation exceeds breakeven inflation over the lives of the bonds, then inflation-linked bonds would provide superior returns to a similar conventional bond. Conversely, if actual inflation is less than breakeven inflation, conventional bonds would provide superior returns to a similar inflation-linked bond. If actual inflation is equal to breakeven inflation, then an investor would be indifferent between both of them.

5.2 Investment characteristics

Some of the characteristics of inflation-linked bonds are equivalent to that of conventional bonds, such as time to maturity, coupon payment frequency, etc. This section considers the key choices and constraints that face the designers of inflation-linked bonds and that are specific for this asset class.

5.2.1 Reference price index

The first task is to determine how realized inflation is measured. Cash flows are linked throughout the life of the bond to changes in a specified price index. Inflation-linked bonds are usually linked to the national consumer price index. The protection an investor attains depends on the correlation between the basket used to form the index and the investor's own consumption.[24] The benefits of consumer price indices are that they are known to the general public, that they are published monthly and that their calculations are reliable.

5.2.2 Indexation

Inflation-linked bonds are designed to protect investors against the erosion of the bonds' cash flows due to inflation, but this protection can take a variety of forms.

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[22] Inflation-linked bonds are priced on the market, and this market is less liquid than the market for conventional bonds, so a liquidity risk premium is demanded for these bonds. Thus, both inflation-linked and conventional bonds exhibit a risk premium. Assuming both premiums are of similar value; i.e., that they compensate each other, then they can be ignored for the calculation of the breakeven inflation.

[23] This formula is also an approximation, the actual formula is $Breakeven\ Inflation = \frac{(1+Nominal\ \ Yield)}{(1+Inflation\text{-}linked\ Yield)} - 1$

[24] Countries from the European Union use the European Harmonised Index of Consumer Prices, formed with household costs across different European countries, instead of their own consumer price index, to increase liquidity. Bonds that are linked to the same reference price index can be traded more easily between those countries, and the more tradeable they are, the lower the liquidity premium gets, and therefore the less the country needs to pay to issue them.

The majority of inflation-linked bonds are structured similar to nominal bonds except that the principal and the coupon payments are not fixed, they change following movements of the reference inflation index used, this structure is known as **capital indexed**. In particular, the principal is continually indexed to realized inflation and the coupons are set as a fixed percentage of this value.

The principal indexed in each period t is calculated as:

$$N_t = N_{t-1} + (N_{t-1} \cdot \pi_t) = N_{t-1} \cdot (1 + \pi_t)$$

where:

π_t inflation accrued at time t

N_t principal inflation indexed at time t

Thus, cash flows at each coupon payment t are:

$$CF_t = CR \cdot \mathrm{N}_t$$

where:

CR real coupon rate of the bond

And at maturity T we simply add the indexed principal repayment:

$$CF_T = CR \cdot N_T + N_T$$

Example:
An inflation-linked bond was issued by the German government at January 2009, with a real coupon rate of 3%, face value of EUR 1000, maturing in five years. The inflation during these five years turned out to be 0.8, 1.3, 2, 2 and 1.4, respectively. Which were the cash flows of the bond using the capital indexation structure?

t	Inflation	Principal	Coupon	Cash Flow
1	0.8	1008	30.24	30.24
2	1.3	1021.10	30.63	30.63
3	2.0	1041.53	31.25	31.25
4	2.0	1062.36	31.87	31.87
5	1.4	1077.23	32.32	1109.55

Another popular indexation structure is to only index the coupons, while the principal redemption value remains constant. This structure is known as **coupon indexed**. The coupons are a variable percentage of the constant principal. Thus, the indexed coupon interest rates (ICR) are calculated simply by adding the inflation rate of the period to the coupon rate of the bond. So, at each period t are:

$$ICR_t = CR + \pi_t$$

Thus, cash flows at each coupon payment t are:

$$CF_t = ICR_t \cdot \mathrm{N}$$

Example:
An inflation-linked bond was issued by the German government the first day of 2009, with a real coupon rate of 3%, face value of EUR 1000, maturing in five years. The inflation during these five years turned out to be 0.8, 1.3, 2, 2 and 1.4, respectively. Which were the cash flows of the bond using the coupon indexed structure?

t	Inflation	Principal	Coupon	Cash Flow
1	0.8	1000	38	38
2	1.3	1000	43	43
3	2.0	1000	50	50
4	2.0	1000	50	50
5	1.4	1000	44	1044

5.2.3 Indexation lag

Ideally, bond cash flows would have to be in line with realized inflation. However, this is not possible in practice because the length of time it takes to compile data and perform calculations means that price indexes are published with a delay of a few months. So bond payments are linked to a lagged value of the price index, this is called **indexation lag**. For most countries this indexation lag is currently three months,[25] so a coupon payment of an inflation-linked bond in April is based on the inflation accrued until January.

The indexation lag causes that there is a period at the end of a bond's life when there is no inflation protection at all, counterbalanced by a period of equal length before it is issued for which inflation compensation is paid. Unless both periods have the same inflation rate, the real return of the inflation-linked bond would not be fully invariant to inflation. This problem is more relevant the longer the indexation lag and the shorter the bond's time to maturity.

Example:
As in the previous examples, assume an inflation-linked bond that was issued in January 2009, maturing in 5 years. If the indexation lag was three months, the annual cash flows would be based on the inflation accrued from October to September, both included. So, for the three months from October to December 2013, there is no inflation protection, compensated by the protection for the period from October to December 2008.

5.2.4 Deflation floor

Most inflation-linked bonds include a deflation floor, a guarantee that an inflation-linked bond's principal repayment is never less than the original par amount. So, if at maturity, the indexed principal is below par the investor will receive the original principal amount. Thus,

$$Principal\ \ repayment\ \ at\ \ maturity = Maximum\ \ (Par\ \ Value,\ \ N_T)$$

Generally, only the principal repayment at maturity is protected against deflation. That is, if during the life of the bond the price index falls below its value at the bond issuance, coupons will be paid off on sub-par principal. Australia is the only important market where coupons are also protected against deflation.

25 In the U.K., prior to 2005, the indexation lag was eight months.

Example:
A one-year capital indexed inflation-linked bond with an annual real coupon rate of 3% is issued with a face value of EUR 1000. What would be its payoff at maturity if inflation turns out to be -1%?

Principal indexed at maturity:

$$N_T = 1000 \cdot (1 - 0.01) = \text{EUR } 990$$

So, the payoff would be:

$$CF_T = 3\% \cdot 990 + \max(1000,\ 990) = \text{EUR } 1029.7$$

A deflation floor can be thought of as a put option embedded in the bond, which costs investors a certain amount. At issuance this put is at par, and the more inflation accrued during the life of the bond, the further out-of-money it is. As long periods of deflation are rare, being Japan the exception in recent times, the longer the maturity of an inflation-linked bond, the less likely the principal would be sub-par at maturity, therefore the lower the embedded price of this option.

Currently, most of the countries include a deflation floor on their inflation-linked bonds, exceptions being for example the U.K. and Canada. Japan includes a deflation floor in its issuances since 2013.

5.2.5 Dual duration[26]

Duration is a measure of the average time for which capital is tied up in a bond and also reflects how sensitive its value is to changes in interest rates. The duration for inflation-linked bonds can be calculated in the same way as for nominal bonds, but needs to consider the indexation of the coupons and the principal. Capital indexed inflation-linked bonds tend to have higher durations than comparable bonds as principal repayment at maturity is usually larger due to its indexation and as the early coupons of inflation-linked bonds are smaller than those of traditional bonds.

Example:
A bond with a 10-year maturity pays a real annual coupon rate of 6%, and has a face value of EUR 100. The annual expected inflation equals 2%. Its yield to maturity is k=10%. What is its Macaulay duration using the coupon indexed structure and the capital indexed structure?

[26] For a mathematical approach of this concept, see Siegel and Waring (2004).

First, consider the coupon indexed bond:

T (Years) [1]	Principal	Coupon	Cash flow CF [2]	PV (CF) $[3]=[2]/(1+k)^t$	CF weight [4] = [3] / Price	Time weighted by CF weight [5] = [1] · [4]
1	100	8	8	7.27	0.0829	0.083
2	100	8	8	6.61	0.0754	0.151
3	100	8	8	6.01	0.0685	0.206
4	100	8	8	5.46	0.0623	0.249
5	100	8	8	4.97	0.0566	0.283
6	100	8	8	4.52	0.0515	0.309
7	100	8	8	4.11	0.0468	0.328
8	100	8	8	3.73	0.0425	0.340
9	100	8	8	3.39	0.0387	0.348
10	100	8	108	41.64	0.4747	4.747
			Price:	**87.71**	**Duration:**	**7.04**

The duration of the bond is 7.04 years. It is easy to see, that this structure is equivalent to a nominal bond that pays an 8% annual coupon.

Second, consider the capital indexed bond:

T (Years) [1]	Principal	Coupon	Cash flow CF [2]	PV (CF) $[3]=[2]/(1+k)^t$	CF weight [4] = [3] / Price	Time weighted by CF weight [5] = [1] · [4]
1	102	6.12	6.12	5.56	0.0636	0.064
2	104.04	6.24	6.24	5.16	0.0589	0.118
3	106.12	6.37	6.37	4.78	0.0546	0.164
4	108.24	6.49	6.49	4.44	0.0507	0.203
5	110.41	6.62	6.62	4.11	0.0470	0.235
6	112.62	6.76	6.76	3.81	0.0436	0.261
7	114.87	6.89	6.89	3.54	0.0404	0.283
8	117.17	7.03	7.03	3.28	0.0375	0.300
9	119.51	7.17	7.17	3.04	0.0347	0.313
10	121.9	7.31	129.21	49.82	0.5691	5.691
			Price:	**87.54**	**Duration:**	**7.63**

The duration of the bond is 7.63 years, which is higher than for the coupon-indexed bond and a comparable nominal bond.

However, this does not mean that the risk of price changes is higher for capital indexed inflation-linked bonds than for nominal bonds, because the duration of nominal bonds is calculated with nominal coupons, while duration of inflation-linked bonds is calculated with real coupons. Therefore, it is not possible to compare both durations. To do this comparison, it has to be taken into account that there are two types of durations, a concept known as dual duration. Real rate duration is the sensitivity to changes in the real rates, while inflation duration is the sensitivity to changes in inflation.

Nominal bonds are sensitive to both changes, therefore for nominal bonds:

$$\textit{nominal bond duration} = \textit{real duration} = \textit{inflation duration}$$

But, inflation-linked bonds are protected against price movements, so a change in inflation will not affect the market value of the bond, therefore:

$$\textit{inflation-linked bond duration} = \textit{real duration}$$
$$\textit{inflation duration} = 0$$

The best way to compare the duration of conventional and inflation linked bonds is to make a clear separation of the two types of duration calculations. The best tool for these calculations is to use key rate durations.[27]

5.3 Market situation[28]

Although the first ever inflation-link bond was issued back in 1780,[29] the market has only been growing strongly over the last ten years. In 1981, the United Kingdom was the first industrialized country to supplement its government bond issue with inflation-linked bonds. The United States with its first issuance in 1998 or Germany in 2006 are further examples of how this asset class has expanded in recent years. As of 2014, all of the G7 countries are issuing inflation-linked bonds and the number of countries issuing these securities is currently expanding with India in 2013 and Spain in 2014 being the most recent countries to join. The total market value of inflation-linked bonds issued worldwide currently amounts to around USD 2.7 trillion (see Figure 5-1), being the U.S., the U.K. and Brazil the leading countries by market value.

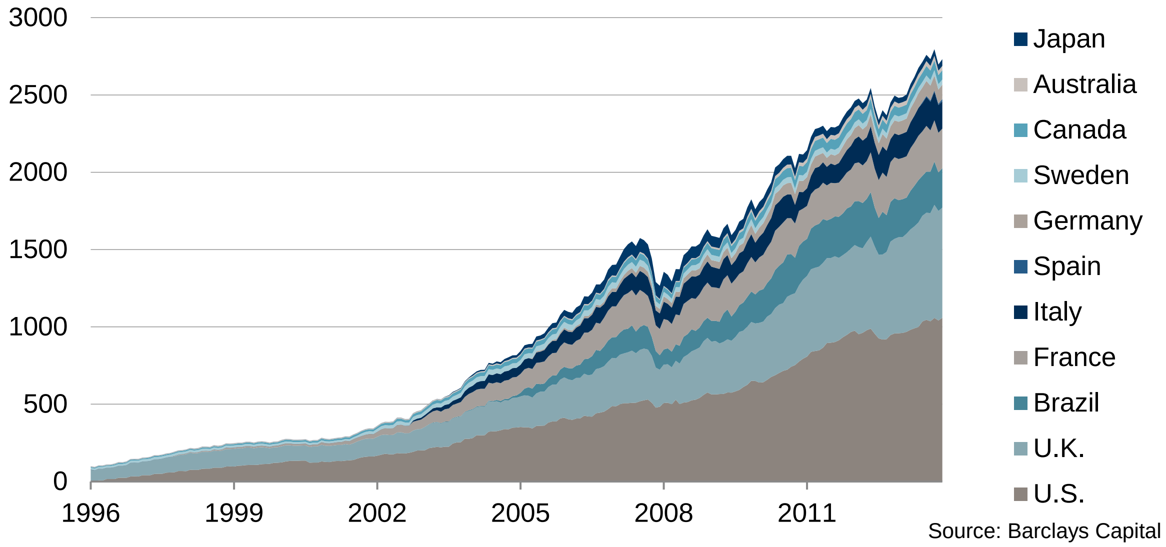

Figure 5-1: Evolution of the market of inflation-linked bonds

27 See the module about “Interest rates - term structure and applications”.

28 Most inflation-linked bonds are issued by countries, only a very small number is issued by companies. Thus, we concentrate on government bonds in this section.

29 By the state of Massachusetts due to inflation caused during the American War of Independence.

Prior to 1981, inflation-linked bonds were only issued by countries that were experiencing episodes of high inflation, such as Brazil and Argentina in the 1950s and 1960s, because it was practically the only way to finance their debt over the long term. The reason was that investors wished to protect their purchasing power against the erosion of the currency's value. What are the reasons for the recent growth of this market, especially for countries where inflation seems to be under control?[30]

- The most cited explanation is to reduce the borrowing costs. This can be achieved in two ways. First, by issuing inflation-linked bonds, countries can avoid paying the inflation risk premium found on conventional bonds. Second, if the government believes that future inflation will be lower than that implied by the market, then it will expect to reduce its debt refinancing costs through inflation-linked bonds.[31]
- To reach new investor groups who are attracted to inflation-linked bonds, because, for example, their liabilities are linked to the cost of living; i.e., such as pension funds.
- To help bolt confidence in the government's fight against inflation, because the state cannot reduce the real cost of borrowing through inflation. This reason is particularly important for emerging countries.

[30] For a more complete analysis see Deacon, Derry and Mirfenderesky (2004), chapter 4.

[31] This latter reason was an important one for the issuance of the U.K. in 1981.

